Accuracy of Six Interpolation Methods Applied on Pupil Diameter Data

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***Abstract*—Eye-tracking is a method of recording the location of gaze as well as pupil diameter (dilation) during active visual behavior. Due to blinking and noise in the recording system, these signals are often briefly "lost", leading to missing data. Here, we aim to analyze the accuracy of six interpolation methods to complete missing values from pupil diameter data. Data interpolation is a type of estimation that constructs new data from existing, neighboring values. Having the possibility to choose from different types of methods, the question is which interpolation method is the most suitable for pupil diameter data. Here, we applied the linear method, the previous neighbors method and four cubic interpolations. Using real data recorded during a visual eye-tracking experiment, we compared the maximum deviations of each interpolation method. Results indicate that for pupil diameter data, the most accurate reconstruction of pupil loss is obtained by Akima, Makima, and the Piecewise Cubic Hermite Interpolating Polynomial.**

Index Terms—Pupil diameter, interpolation, missing data, eye-tracking

I. Introduction

In an ever-increasing digitization age, collecting datasets during visual experiments is more and more often, including data from eye-tracking measurements. However, sub-optimal experimental setup or biological related events, such as body movements, interfere with the optimal recording of signals and cause data loss. This may require the manipulation of data in order to restore missing information by an approximation function [1], that estimates values between given data points [2]. Such estimation is called interpolation. Here, we applied several methods of interpolation to reconstruct time-series data representing the pupil size, offering an indication of how pupil diameter changes over time.

The change of pupil diameter over time is important because it is modulated by the presentation of visual stimuli and it is also a signature of cognitive workload engaged during task performance. This relation has long been documented [3] and it has been suggested that the time course of pupil dilation and contraction correlates with the retrieval of memory [4], attention, decision-making, or emotions [5].

Visual experiments using eye-tracking technology offer the possibility of exploring task-evoked pupil diameter time-series. The raw pupil data signal, as is captured by the eye-tracking system, represents the actual physical size of the pupil [6], which is then converted into a standard format [7] that contains series of values slowly fluctuating over time. The usual pipeline for preprocessing pupil data includes i) filtering the raw data in order to extract valid samples by removing samples that are the result of artifacts or noise, ii) smoothing the valid samples, and iii) applying a baseline correction [7].

Due to several factors, such as blinking, movement, or noise, recorded eye-tracking signals frequently exhibit what is called "pupil loss", leading to missing data in the pupil diameter time series. The reconstruction of the signal by applying an interpolation method is crucially important since one of the main features of the pupil size is its continuity over time. Another characteristic of the pupil signal is its slow variance over hundreds of milliseconds / seconds, with small signal noise on top caused by rapid moving changes in the course of milliseconds [8]. The slow pupil size changes are caused by a muscle involved in the constriction and dilation that contains smooth muscle fibers [9]. When applying an interpolation method to reconstruct the missing data samples, we have to take into account these signal characteristics. Thus, often, a smoother interpolation method is desirable, which offers a smooth transition between adjoining data points [10].

The goal of this investigation was to assess the accuracy of several interpolation methods to discover the one that is the most suitable for pupil diameter data, considering the latter's characteristics. Next, we focus on related work, with a short presentation of the interpolation methods chosen.

II. Related Work

A. Pupil diameter and the interpolation of missing data

Pupil diameter data is represented as a continuous signal over time. Therefore, missing samples (e.g. caused by blinks) and a noisy signal [11] represent a potential source of error in data processing [12]. This is caused by the fact that these interferences do not represent an on-off event determined by

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the visual stimuli [13]. Even though the standard pipeline of preprocessing pupil data addresses this kind of issues, and the reconstruction of missing samples is usually performed by applying interpolation, there is no universally accepted interpolation method considered most suitable for pupil size data. To the best of our knowledge, there is only one study that particularly addresses the issue of missing data from pupil size records: Mathôt's work, reconstructing missing data caused by blinks with cubic-spline interpolation [14]. In general, the most frequently used methods in the investigation of pupil size are the cubic and linear ones [15].

B.1. Interpolation methods

When choosing an interpolation method, one has to consider in addition to the characteristics of the data on which the interpolation has to be applied, also the method's characteristics. Therefore, next, we will shortly describe several known methods of interpolation, and provide a critical analysis in terms of complexity and smoothness. We will focus on the following methods: Previous neighbor [16], Linear [2], Cubic Spline [16], Piecewise Cubic Hermite Interpolating Polynomial (Pchip) [16], Akima [17], and Makima [16]. These were chosen for the complexity differences of their algorithms given by the type and the degree of the polynomials used and the order of the derivatives. Also we chose these methods because they represent different types of interpolation (replication, linear, and cubic type [10]).

B.2. The complexity of the algorithms

The simplest interpolation methods are Previous neighbor and Linear interpolations. The Previous neighbor interpolation is a replication method. It identifies the previous data value and assigns it to the interval of unknown values (replicate the value). Therefore, the interpolated values are entirely determined by the known previous value [16]. The Linear Interpolation method is almost as simple, and it works by estimating new data points by joining with a straight line the nearest known values located to the left and right, using piecewise linear polynomials [2].

The next interpolation methods are based on cubic piecewise polynomials. The Cubic Spline interpolation uses cubic piecewise polynomials named splines, using not-a-knot end conditions. This method estimates new data points by joining the known values with a cubic polynomial, based on the values at neighboring grid points [16]. The Piecewise Cubic Hermite Interpolating Polynomial (Pchip), as its name suggests, uses piecewise cubic Hermite spline interpolating polynomials, P(x), which use a local scheme, where each point is determined independently for a given data subinterval, $x_k \leq x \leq x_{k+1}$. At the interpolation points, P(x) has derivatives (slopes) computed strictly from the data, respecting data monotonicity. Therefore, P(x) is monotonic where data points are also monotonic and, where the data points have a local extremum, so does P(x) [16]. Akima interpolation method is also based on cubic polynomials, and it uses only the next neighbor values to estimate the new data points. Makima interpolation uses modified Akima algorithms, based on piecewise cubic Hermite interpolation [16].

The difference between the algorithms is also underlined by the degree of the polynomials and the order of the derivatives. Linear interpolation uses polynomials of first degree, to approximate data points [18]. The Previous neighbor method uses splines of degree 0, also referred to as piecewise constant functions, which simply assign a constant value to any interval [19]. The Cubic Spline, Pchip, Akima and Makima interpolations use third degree polynomials. Related to the order of their derivatives, the Linear has polynomials with zero-order derivatives, the Previous Neighbor method has discontinuous derivatives, the Cubic Spline has continuous second-order derivatives, Pchip has first-order derivatives and Akima/Makima has continuous first-order derivatives.

B.3. Critical Analysis of the considered methods

The Previous neighbor is the fastest interpolation concerning the computation time. However, it may not be suitable for pupil diameter data, since it is unlikely that the missing values are always the same as the previous known value. Even though the Linear method is almost as easy and fast as the Previous neighbor, and requires low memory to be performed, it determines approximations that are not very smooth, and it shows roughness at merging points, which may be a problem [16].

Compared with the former, cubic interpolations require more computation time and memory, however, produce smoother curves. The Spline method produces smooth curves, with minimal variation, due to the continuity properties of this interpolation method. Also, the method has smaller errors compared with other interpolation polynomials (without undesirable oscillations between the interpolation points) [16]. Pchip is known for the fact that it is shape preserving. This means that the slopes preserve the shape of the known values, respecting monotonicity presented in the original data [20]. It produces results without overshoots and with few oscillations for non-smooth data. Akima also preserves the slope of the known values [16]. Therefore, the resulting curve will appear natural and smooth [17]. It allows fewer undulations of low amplitude compared to the Cubic Spline interpolation, however, it is not as aggressive in their reduction as the Pchip interpolation [16].

Makima interpolation uses modified Akima algorithms that offer more weight to the edges where the slope is near to zero, eliminating overshoot and undershoot when the data is constant, thus increasing robustness. Regarding undulations, Makima gives intermediate results between Spline and Pchip interpolations. Therefore, it has wiggles that are lower in amplitude compared with Spline, but it is less aggressive at reducing wiggles compared with Pchip [16].

As we have shown, multiple interpolation methods can be applied, with various effects on the resulted data and different degrees of complexity. Unfortunately, there is no objective measure as to what kind of interpolation method is the best for pupil diameter data. Next, we will present the issue of missing values from pupil size data recorded during a visual experiment and will assess the above-mentioned interpolation methods' behavior on such data to discover the one that is the most suitable.

III. DATA COLLECTION APPROACH

The recorded data was obtained during an eye-tracking experiment realized with the purpose of investigating visual perception and the strategies applied in visual recognition in human subjects. The experiment was performed on 11 subjects. All of them had normal or corrected-to-normal vision. The visual stimuli (n = 180) were presented on a LCD monitor (Samsung 2233RZ). They were generated by the "Dots" technique [21]. This stimuli generation technique enables prolonged visual exploration. After the presentation of each stimulus, subjects explore visually and, when they reach a decision, they verbalize what they have seen. Each such time interval, spanning the stimulus presentation and response, is called a trial. We thus obtained 180 trials corresponding to the 180 presented stimuli. During each trial, eye-tracking measurements were performed with an ASL EyeStart 6000 system, to record ocular movements that give information about the eye movement patterns and the pupillary response. A chin rest was used to ensure head stability and the viewing distance. The sampling rate of the eye-tracker was 50 Hz.

The pupil diameter signal was preprocessed using an in-house developed software [21]. The preprocessing consisted in data cleaning and filtering, including blinks and impulse and ramp noise removal. Pupil diameter data was analyzed using a pre-stimulus baseline correction. Pupillary response was measured as the difference between pupil peak and pupil baseline. Data was then imported in Matlab for processing analysis such as evoked pupil diameter analysis. During pupil diameter processing, for some trials, we observed pupil losses that often appear as spikes in the data. Although off-line filtering was performed to eliminate some recording imperfections, the filters marked the samples with pupil loss as missing data (Fig. 1).

Fig. 1. Example trial with missing data.

IV. ANALYSIS OF DATA AND INTERPOLATION RESULTS

In order to discover which interpolation method best suits our data, we have taken several steps. First, we analyzed the distribution of the missing data in trials, i.e., the duration and the position distribution of the missing data segments. These distributions are illustrated in the histograms presented in Fig. 2. To obtain test data with known "ground truth", we then chose data trials without missing values and automatically replace existing, known values with zeros (representing missing data), according to the original distribution of missing data. In this manner we intend to compare the results of the interpolation methods applied on the generated test data, containing missing values artificially inserted, with the "gold" standard, i.e. the original data set.

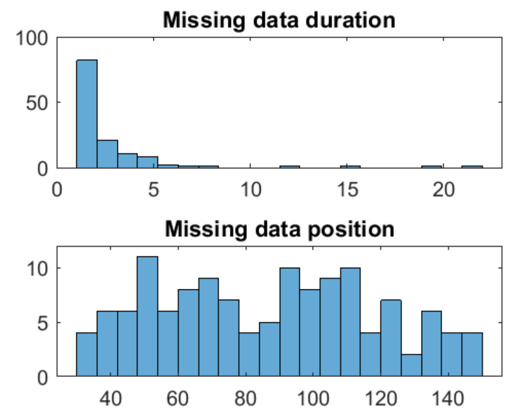

Fig. 2. Distributions of duration and position for missing data segments.

The interpolations were applied on a trial-by-trial basis in order to estimate the pupil diameter, and were realized in Matlab using built-in functions. The interpolation functions return interpolated values at specific query points (time points with missing values where the interpolation is needed), using the known values according to the algorithms of each method. Fig. 3 illustrates the reconstruction of a single trial:

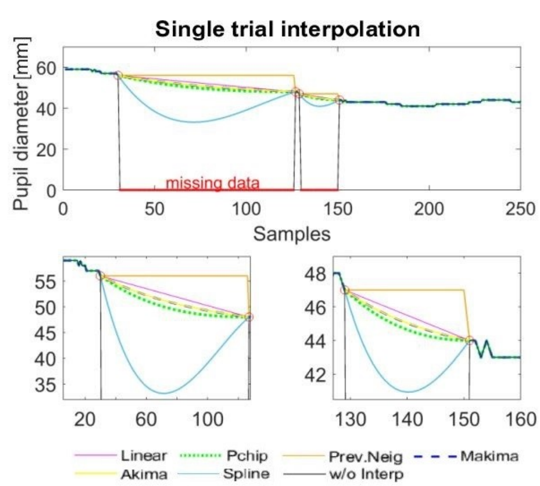

Fig. 3 Reconstruction of a signal with artificially inserted missing data, using the interpolations. Bottom: zoom on the interpolated areas.

In order to compare the interpolations, we have measured the difference between the interpolated data and the original data. This was realized by calculating the Maximum Deviation for each interpolation method, which offers information (high bound) about how close the interpolated values are to the original ones. We have applied the following Maximum Deviation formula:

$$Maximum\ Deviation = \max\big(abs(Interpolated\ Data - Real\ Data)\big) \quad (1)$$

We applied the above steps multiple times, with different durations and positions of missing data, considering the distribution of the missing values from the original data. We measured the Maximum Deviation for each such test. On a visual inspection of the results, illustrated in Fig. 4, the differences between the interpolation methods are obvious, indicating that the poorest performing method was the Spline interpolation, having the largest Maximum Deviation. This was followed by the Previous Neighbor method and the Linear method. The Akima, Makima, and Pchip methods had the lowest Maximum Deviation. Similar results were obtained when we used Root Mean Squared Error (RMSE).

In order to decide if a parametric or a non-parametric test is more suitable to determine the statistical significance of differences between interpolation methods we applied the Shapiro-Wilks normality test. The Shapiro-Wilks results indicated that p-value = 0 < α, thus it is assumed that the data is not normally distributed. The Skewness Shape was asymmetrical, right/positive skew, long right tale. The W value was 0.7501, thus not in the 95% critical value accepted range: [0.9950:1.0000]. However, since a perfect symmetrical distribution with skewness of zero is almost never seen in real data, and considering that the skewness level was around zero, we decided to treat the data as normally distributed and we applied a parametric test to analyze the Maximum Deviation results.

To determine which of the observed differences in Maximum Deviation were statistically significant, we next applied an analysis of variance (ANOVA) test. ANOVA revealed that differences between methods in terms of Maximum Deviation were significant (p value = 1.1102e-16, < .05 = α; F value = 3361.551 > F statistic = 2.2291). The observed effect size f was large (5.31), indicating that the magnitude of the difference between the averages was large. According to a Tukey Kramer test, the means of the following method pairs were significantly different: Akima – Spline, Akima – Linear, Akima – Previous Neighbor, Makima – Spline, Makima – Linear, Makima – Previous Neighbor, Pchip – Spline, Pchip – Linear, Pchip – Previous Neighbor, Spline – Linear, Spline – Previous Neighbor, Linear – Previous Neighbor. The same results were obtained when we applied a set of Bonferroni corrected t-tests.

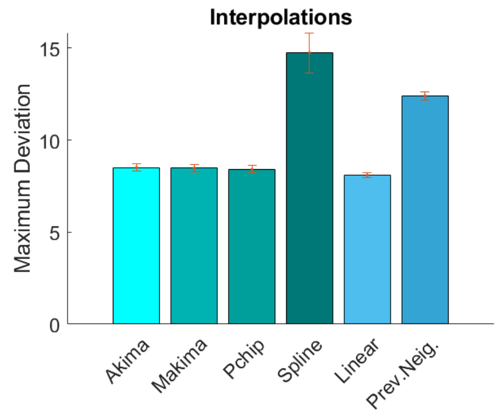

Fig. 4 Maximum Deviation results as a function of different interpolation methods. Error bars represent s.d.

Concerning the comparison between Akima, Makima and Pchip, none of the statistical tests indicated a significant difference between their averages. Since these three were the interpolations with the smallest Maximum Deviation, we can conclude that these are the most accurate ones, offering the best fit.

V. DISCUSSIONS

We have compared six interpolation methods to assess their accuracy in approximating new data points in pupil diameter data. The methods were chosen from three types of interpolation: replication, linear, and cubic type, and they differed in terms of the complexity of their algorithms and the known advantages and disadvantages. Respectively, even though the replication and the linear types are easy, fast and require low computation memory, they show roughness at merging points. By contrast, the cubic interpolations, even though it requires more memory and computation time, it produces smoother curves.

According to the results of this investigation, Akima, Makima, and Pchip interpolation methods yield the smallest Maximum Deviation, and Cubic Spline and Prev. Neig., the largest ones. Among the cubic interpolation types, Akima, Makima, and Pchip are known for their time and memory efficiency and the smoothness of the curve fitting, which makes them suitable for smooth data that vary slowly, such as pupil data. By comparison, the other cubic interpolation, Cubic Spline, also known for its smoothness, creates jumps between data points, resulting in larger errors. Related to the Linear method, this is known for the fact that it may produce curves that are not very smooth. For pupil diameter data this seems to significantly affect the accuracy of the interpolation. A possible explanation is the slow variation of the pupil diameter data.

In light of present results, Akima, Makima, and Pchip are more advisable to be used for the reconstruction of missing values from pupil diameter data. This is consistent with other investigations, where missing values from pupil size records were reconstructed using cubic interpolation. However, even though the Linear interpolation is used in many other investigations [15], our results indicate that better methods are available, such as Akima, Makima, or Pchip. From among the cubic type methods, the spline interpolation is the least recommended. We argue that a suitable interpolation method has to be considered for pupil diameter data, taking into account its continuity and slow variation over time.

Future research may take into account the augmentation of the built-in functions from Matlab used for interpolations. For example, to increase the accuracy of curve reconstruction and to deal with numerical interpolation and differentiation, the built-in function Pchip, which was applied in the current study, should be modified in order to allow to explicitly specifying the derivatives that preserve monotonicity.

ACKNOWLEDGMENTS

This work was supported by a grant from the Romanian National Authority for Scientific Research and Innovation, CNCS-UEFISCDI (project COFUND-NEURON-NMDAR-PSY).

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